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Non-Markovian electron diffusion in the auroral ionosphere at high Langmuir-wave intensities

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Abstract. A generalized non-Markovian diffusion model which describes the interaction of Langmuir waves with field-aligned electrons in the auroral ionosphere is found to be relevant to the large amplitude Langmuir waves measured by UC Berkeley sounding rockets and by the Freja satellite. This model is valid for any ordering of the diffusion time and the autocorrelation time (the standard quasilinear diffusion model requires the diffusion time to be much longer than the autocorrelation time). We demonstrate that, while the quasilinear diffusion approximation is valid for the lower altitudes studied by the Berkeley sounding rockets, the non-Markovian model is needed for the intense waves observed at the higher altitudes probed by Freja. A test particle simulation is employed to quantify our estimates of the relevant timescales.

Introduction

Intense bursts of Langmuir waves are commonly detected in the auroral ionosphere by *in situ* spacecraft. Over the past decade, the sounding rockets of the University of California at Berkeley have observed large amplitude Langmuir waves ($E \approx 50 - 500$ mV/m) correlated with precipitating, energetic, field aligned electrons (100 eV to 3 keV) at altitudes near 700 km [*McFadden et al.*, 1986; *Ergun et al.*, 1991]. The Freja satellite has made similar measurements at higher altitudes (1700 km) [*Kintner et al.*, 1995; *Stasiewicz et al.*, 1996].

While the wave spectrum has been modeled in detail for a fixed bump-on-tail instability [*Newman et al.*, 1994], the effect of the waves on the distribution has yet to be determined self-consistently. Although the acceleration of ions by lower hybrid waves in the auroral ionosphere has been modeled extensively at the level of quasilinear diffusion [*Retterer et al.*, 1994], the interaction of Langmuir waves with electrons has not.

The standard model for wave-particle interactions, known as quasilinear diffusion, describes the diffusion of particles in velocity space due to a spectrum of randomly phased waves. The quasilinear model is *Markovian* in the sense that the evolution of the particle distribution function depends only on its present value, and on none of the preceding values [*Stratonovich* 1963].

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Quasilinear theory might appear to be a reasonable candidate for modeling Langmuir wave-particle interactions in the auroral ionosphere since previous studies have found the saturated waves to be phase incoherent [*Newman et al.* 1994]. However, in addition to phase incoherence, there is a timescale ordering which must be satisfied. The quasilinear diffusion approximation is only valid if the particle diffusion time is much longer than the wave autocorrelation time. If this is not the case, the evolution of the particle distribution will be non-Markovian.

In this letter we assess the validity of quasilinear diffusion in the auroral ionosphere, find it to be inadequate, and present a model of non-Markovian diffusion valid in the regime where the quasilinear approximation breaks down. First we estimate the relevant timescales and show that quasilinear diffusion is not valid for the very high wave intensities reported by *Kintner et al.* [1995] and by *Stasiewicz et al.* [1996] at higher altitudes in the auroral ionosphere. We then quantify our estimates using a test particle simulation. Finally, we discuss the application of the non-Markovian model to the higher alititude case.

Quasilinear diffusion in a uniform magnetic field

The standard magnetized quasilinear equation (e.g. *Shapiro and Shevchenko* [1962], *Kennel and Englemann* [1966]) for collisionless electron diffusion due to electrostatic waves is:

$$\partial_t F(\mathbf{v},t) = \partial_{\mathbf{v}} \cdot \mathbf{D}(\mathbf{v},t) \cdot \partial_{\mathbf{v}} F(\mathbf{v},t), \qquad (1)$$

where $F(\mathbf{v},t)$ is the spatially averaged (over volume V) particle distribution. The quasilinear diffusion tensor, $\mathbf{D}(\mathbf{v},t)$, is

$$\mathbf{D}(\mathbf{v},t) = \lim_{V\to\infty} \frac{4\pi^2 e^2}{v_z V m_e^2} \sum_{n=-1}^{1} \int_0^\infty k_\perp dk_\perp |\mathbf{E}_\mathbf{k}(t)|^2 J_n^2(b) \frac{\mathbf{a}_n \mathbf{a}_n}{k^2}. \qquad (2)$$

Here, ω_{ce} is the electron cyclotron frequency, J_n are the Bessel functions, and $b = k_\perp v_\perp/\omega_{ce}$. $\mathbf{a}_n$ is defined by $\mathbf{a}_n = (n\omega_{ce}/v_\perp)\hat{e}_\perp + k_z\hat{e}_z$, where $\hat{e}_\perp$ and $\hat{e}_z$ are unit vectors in the directions perpendicular and parallel to the magnetic field, respectively. $\mathbf{a}_n$ and k_z are evaluated for the resonance condition, $k_z v_z = \omega_r - n\omega_{ce}$. We assume a gyrotropic wave spectrum. For the auroral ionosphere, we assume $k_\perp^2 R_e^2 \ll 1$, where $R_e \equiv v_e/\omega_{ce}$ is the Larmor radius of a thermal electron and v_e is the electron thermal velocity. When we calculate the diffusion tensor components, we find the perpendicular components to be exponentially small compared to the parallel components.

The derivation of (2) requires that the system have a wide diffusive regime so the initial evolution of the particle distribution due to coherent acceleration can be neglected compared to its subsequent diffusive behavior. This gives rise to the condition:

$$t_D \gg t_{ac}, \tag{3}$$

where t_D is the spectral diffusion time and t_{ac} is the linear wave autocorrelation time.

The autocorrelation time or phase-mixing time is defined by $t_{ac} \equiv 1/k_z \Delta v_{\phi,z}$, where $\Delta v_{\phi,z} = \Delta(\omega/k_z)$ is the phase velocity half-width of the wave spectrum in the $\hat{v}_z$ direction (i.e. parallel to the magnetic field) and k_z is a typical wavenumber [*Cary et al.*, 1992]. For times much less than t_{ac} (the free streaming limit), particles are accelerated by a coherent field, even though the spectrum consists of randomly phased waves. The spectral diffusion time, $t_D = (\Delta v_{\phi,z})^2/2D_{ql}$, is the time it takes for a particle to diffuse quasilinearly across the half-width $\Delta v_{\phi,z}$ of the wave spectrum. D_{ql} is the dominant component of the diffusion tensor for a typical value of particle velocity, assuming the quasilinear diffusion approximation is valid.

Non-Markovian diffusion in one dimension

We now present a model for non-Markovian diffusion in one dimension, valid when (3) is not satisfied. We will later verify our 1-D assumption using test particle simulations. By taking the spatial average of the one-dimensional Vlasov equation, a non-Markovian diffusion equation is obtained:

$$\partial_t F(v,t) = \partial_v \int_{t_0}^{t} \tilde{D}(v,t,t')\partial_v F(v,t')\omega_{pe} dt', \tag{4}$$

$$\tilde{D}(v,t,t') = \lim_{L\to\infty} \frac{e^2}{2\pi L \omega_{pe} m_e^2} \int E_k^*(t) E_k(t') e^{-ikv(t-t')} dk, \tag{5}$$

where t_0 represents the initial time. Although equation (4) is an intermediate step in the standard quasilinear diffusion derivation (e.g., *Aamodt and Drummond* [1964]), this equation has, to our knowledge, never previously been interpreted physically, nor suggested as a means to calculate the evolution of the particle distribution. We have neglected terms of higher order in $E(z,t)$, which correspond to additional nonlinear effects such as trapping. The conditions for making this approximation were verified via test particle simulations. In the event that the timescale for the evolution of $F(v,t)$ is much larger than the autocorrelation time we recover the standard quasilinear result.

Xia, et al. [1993] have studied one-dimensional non-Markovian diffusion from a stochastic differential equations perspective by obtaining a Langevin equation for the stochastic velocity variable and defining the non-Markovian diffusion coefficient as the time rate of change of the velocity dispersion. However, (4) is more readily usable for calculating the evolution of the particle distribution function.

Application to the Auroral Ionosphere

Timescale estimates

We use parameters relevant to the auroral ionosphere to estimate t_{ac} and t_D for two altitudes (700km and 1700km) in order to determine where quasilinear theory is valid. The wave spectrum we use in our estimates is motivated by the two-dimensional wave-wave simulations in figure 5 of *Newman et al.* [1994], where magnetized Langmuir waves were assumed to be excited by an electron beam with beam velocity $v_b \approx 28 v_e$. Although the Zakharov simulations were performed for parameters characteristic of the lower altitude, test particle simulations reveal that the parallel diffusion of electrons depends almost exclusively on the parallel k-space distribution of the wave spectrum. This finding is consistent with the fact that the relevant timescales, t_D and t_{ac}, depend only on the parallel properties of the spectrum, but not on the perpendicular properties. Since the altitude dependence of the Langmuir spectrum is reflected primarily by its perpendicular width (due to the change in the value of ω_{ce}/ω_{pe}), we are justified in using the same model spectrum at the higher altitude as well. It is true that the cyclotron damping and magnetic dispersive corrections are sensitive to ω_{ce}/ω_{pe}. However, these factors only affect the perpendicular width of the spectrum.

We fit the saturated turbulent wave spectrum with a gaussian of the form

$$|\mathbf{E}(\mathbf{k})| = A e^{-(k_z - \langle k_z \rangle)^2/2(\delta k_z)^2} e^{-(k_\perp - \langle k_\perp \rangle)^2/2(\delta k_\perp)^2}, \tag{6}$$

where A is normalized such that $|E|_{rms} = 500$ mV/m, representing the largest amplitude Langmuir waves observed by Bidarca [*Boehm*, 1987]. Wave amplitudes of this size were also observed by Alaska '88 [*Ergun et al.*, 1991], and Freja [*Kintner et al.*, 1995]. Equation (6) describes a bi-gaussian spectrum centered around mean wave vector $(\langle k_z \rangle, \langle k_\perp \rangle)$, with parallel and perpendicular widths δk_z and $\delta k_\perp$, respectively. The phase velocity width of the spectrum in the parallel direction is $\Delta v_{\phi,z} = 1.8 v_e$. The autocorrelation time for this spectrum is $t_{ac} = 1/k_z \Delta v_{\phi,z} \approx 16 \omega_{pe}^{-1}$.

At the altitude of 700 km we use $\omega_{ce}/\omega_{pe} = 1.2$, $f_{pe} = 1.2$ MHz, and $T_e = 1$ eV, where $\omega_{pe} = 2\pi f_{pe}$

is the electron plasma frequency and T_e is the electron temperature. These values are consistent with the data obtained by Bidarca. Upon substituting (6) into (2) we find the parallel quasilinear diffusion coefficient for the low altitude to be $D_{zz} \approx 0.025 v_e^2 \omega_{pe}$, giving a quasilinear diffusion time of $t_D = (\Delta v_{\phi,z})^2 / 2D_{zz} \approx 65\omega_{pe}^{-1}$. Condition (3) is therefore satisfied, and particle transport can be accurately described by quasilinear diffusion.

At the higher altitude of 1700 km, the values ω_{ce}/ω_{pe} =3.0, f_{pe} = 284 kHz, and T_e = 1 eV were used, consistent with Freja observations. The parallel quasilinear diffusion coefficient was found to be much larger ($D_{zz} \approx 0.29 v_e^2 \omega_{pe}$) at this higher altitude, which results from the fact that the diffusion coefficient is proportional to the ratio of wave energy to thermal energy ($|E|^2/8\pi n_e T_e$). Although the wave field strengths at both altitudes are similar, the electron density at the higher altitude is an order of magnitude smaller than at the lower altitude. The diffusion time for the higher altitude is $t_D \approx 5.6\omega_{pe}^{-1}$. Since $t_D < t_{ac}$, condition (3) is clearly violated and quasilinear diffusion is not valid. In physical terms, particles with velocities near the spectral peak will be transported (in velocity space) out of resonance with the wave spectrum before significant phase-mixing occurs. Thus, the particles are effectively accelerated by a partially coherent field, in contradiction to the fundamental assumption of quasilinear theory.

While the quasilinear diffusion approximation is not valid at the higher altitude for the field strength of $|E|_{rms}$ = 500 mV/m, it will be valid for sufficiently weaker fields. In order to estimate the field strength at which quasilinear diffusion starts to break down, we note that the quasilinear diffusion time is inversely proportional to the square of the intensity of the Langmuir field. Assuming that the parallel spectral width is similar to the width obtained by *Newman et al.* [1994], waves with a nominal field strength of $|E|_{rms} \approx$ 150 mV/m will have $t_D \approx 64\omega_{pe}^{-1}$, which is comparable to the quasilinear time at the lower altitude. These waves will be treatable with quasilinear theory.

Test particle verification of timescale estimates

A test particle simulation was used to verify the above estimates. The initial distribution of test particles was a narrow gaussian centered around $v_z = 28.1 v_e$ and $v_\perp = 3.29 v_e$, corresponding to the phase velocity of the center of the wave spectrum which was discussed earlier. This ensemble of particles was advanced in time according to the Lorentz force due to the wave spectrum (6) and to the background magnetic field. The diffusion tensor was then calculated as

$$D_{ij} = \frac{\langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle}{2t}, \tag{7}$$

where t is the time at which we measure D_{ij}. Note that for the diagonal elements of the diffusion tensor, (7) reduces to the familiar expression for the diffusion coefficient, $D_{zz} = \delta v^2/2t$, where $\delta v \equiv \langle (v_z - \langle v_z \rangle)^2 \rangle^{1/2}$ is the standard deviation of the test particle distribution, and the brackets $\langle\rangle$ represent an average over parallel velocity. To obtain a meaningful result, care must be taken to calculate the diffusion coefficient at a time t such that $t_{ac} \ll t \ll t_D$.

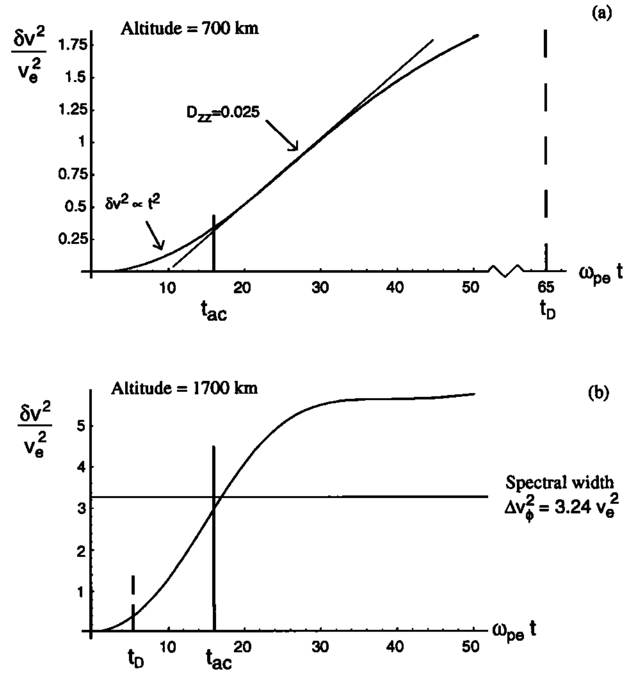

Figure 1. (a) The velocity spread δv of the test particle distribution versus time for the lower altitude (700 km). (b) The velocity spread δv of the test particle distribution for the higher altitude (1700 km).

For both altitudes the time evolution of the perpendicular width of the particle distribution is oscillatory around the initial width, indicating that perpendicular diffusion is negligible. Furthermore, after running our simulations for several different perpendicular spectral widths, we find our results to be independent of the perpendicular spectral shape. This justifies our use of the same spectrum at both altitudes, since the ratio ω_{ce}/ω_{pe} (which depends on altitude) only affects the perpendicular extent of the spectrum.

For the lower altitude, the test particle diffusion coefficient agrees well with quasilinear theory. In figure 1a, we see that for $t \gtrsim 1.5 t_{ac} \approx 24\omega_{pe}^{-1}$ the evolution of the test particle distribution is diffusive ($\delta v^2 = 2D_{zz}t$). The value of the numerical diffusion coefficient is within two percent of the quasilinear result. For $t < t_{ac}$ the quantity δv^2 increases quadratically with time according to $\delta v(t)^2 = (eE_{rms}/m)^2 t^2$. This behavior is consistent with acceleration due to a coherent wave field (the so-called free-streaming limit) to within 1%.

Using the same wave amplitude and spectrum at the higher altitude as the lower altitude, we find the acceleration in the free streaming limit ($t < t_{ac}$) is still within less than one percent of theory. However, for $t > t_{ac}$, we see that no diffusive regime exists (figure 1b), since the test particle distribution width δv has already diffused beyond the wave spectrum width $\Delta v_{\phi,z}$ and reached saturation. These findings are consistent with our previous estimates.

Discussion

Quasilinear theory is not sufficient to describe the evolution of the electron distribution function in the auroral ionosphere. Although the quasilinear diffusion approximation is valid (i.e., $t_D \gg t_{ac}$) at the lower altitudes probed by Bidarca, it breaks down at the higher altitudes probed by Freja for intense field strengths ($E \approx 500\text{mV/m}$). For this case, we have constructed a non-Markovian diffusion model, valid for any ordering of the diffusion and autocorrelation times. While quasilinear theory is Markovian and only requires knowledge of the instantaneous electron distribution and wave spectrum to evolve the electron distribution, the non-Markovian model includes memory effects, which take into account the history of the electron distribution and wave spectrum. The validity condition for quasilinear diffusion (3) can be expressed as $(\Delta v_{\phi,z}^4/v_e^4)\langle k_z\rangle^2\lambda_e^2 \gg |E|^2/8\pi n_e T_e$. Thus, either larger field strengths or narrower regions of unstable waves are more likely to produce non-Markovian diffusion.

Interpretation of non-Markovian diffusion is complicated by the fact that the transport of particles in velocity space depends on the past history of both the particle distribution function and the wave fields. For example, in the case of an unstable bump-on-tail electron distribution, the slope of the distribution decreases as the wave levels rise so that $\partial_v F(v,t')$ in Eqn. (4) is largest for $t' \ll t$ while $\tilde{D}(v,t,t')$ contributes most when $t' \approx t$. One possible consequence of non-Markovian diffusion is overflattening of the distribution (i.e. diffusion persisting after $\partial_v F = 0$, which is inconsistent with quasilinear theory). We have observed examples of transient overflattening in 1-D Vlasov simulations, but a definite association with non-Markovian diffusion will require further study.

We have also verified (using test particle simulations) that both quasilinear and non-Markovian diffusion are effectively one-dimensional (along the magnetic field) for auroral parameters. This is a useful simplification for constructing a future model of Langmuir turbulence in the auroral ionosphere that contains both wave-wave and wave-particle nonlinearities. Since the one-dimensional non-Markovian model is computationally tractable, a self-consistent model could be obtained by coupling one-dimensional non-Markovian diffusion to the two-dimensional nonlinear wave-wave simulations which were used to generate the wave spectrum involved in the above calculations. This model would be valid in the general non-Markovian regime, which includes the free-streaming and quasilinear limits, thereby allowing the study of self-consistent Langmuir turbulence at both high and low altitudes of the auroral ionosphere.

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References

Aamodt, R. E. and W. E. Drummond, Resonant wave-wave scattering of plasma oscillations, *Phys. Fluids, 7,* 1816, 1964.

Boehm, M. H., Waves and static electric fields in the auroral acceleration region, Ph.D. thesis, Dept. of Phys., Univ. of Calif. at Berkeley, 1987.

Cary, John R., Isidoros Doxas, D. F. Escande, and A. D. Verga, Enhancement of the velocity diffusion in longitudinal plasma turbulence, *Phys. Fluids B, 4,* 2062, 1992.

Ergun, R. E., C. W. Carlson, J. P. McFadden, and J. H. Clemmons, Langmuir wave growth and electron bunching: Results from a wave-particle correlator, J. Geophys. Res., *96*, 225, 1991.

Kadomtsev, B. B., *Plasma Turbulence*, Academic Press, London, pp. 15-32. 1965.

Kennel, C. F. and F. Engelmann, Velocity space diffusion from weak plasma turbulence in a magnetic field, *Phys. Fluids, 9,* 2377, 1966.

Kintner, P. M., J. Bonnell, S. Powell, and Jan-Erik Wahlund, First results from the Freja HF snapshot receiver, Geophys. Res. Lett., *22*, 287, 1995.

McFadden, J. P., C. W. Carlson, and M. H. Boehm, High frequency waves generated by auroral electrons, J. Geophys. Res., *91,* 12079, 1986.

Newman, D. L., M. V. Goldman, R. E. Ergun, and M. H. Boehm, Langmuir turbulence in the auroral ionosphere 2. Nonlinear theory and simulations, J. Geophys. Res., *99,* 6377, 1994.

Retterer, J. M., Tom Chang, and J. R. Jasperse, Transversely accelerated ions in the topside ionosphere, J. Geophys. Res., *99,* 13189, 1994.

Stasiewicz, K., B. Holback, V. Krasnoselskikh, M. Boehm, R. Bostrom, and P. M. Kintner, Parametric instabilities of Langmuir waves observed by Freja, Geophys. Res. Lett., *101,* 21515, 1996.

Stratonovich, R. L., *Topics in the theory of Random Noise Volume I,* Gordon and Breach, New York, 1963.

Shapiro, V. D., and V. I. Shevchenko, The nonlinear theory of interaction between charged particle beams and a plasma in a magnetic field, JETP, *15,* 1053, 1962.

Xia, H., O. Ishihara, and A. Hirose, Non-Markovian diffusion in plasma turbulence, *Phys. Fluids B, 5,* 2892, 1993.

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